summerschool UTRECHT

Summer School on Geometry

10–21 August 2026

Last updated: June 12, 2026

Course organisers. Álvaro del Pino Gómez and Wilfred de Graaf.
Contact. w.f.degraaf@uu.nl
For more information and to apply: www.utrechtsummerschool.nl/courses/science/geometry.

Location. Room 611 on the 6th floor of the Hans Freudenthal building, Budapestlaan 6, Utrecht: https://www.uu.nl/en/hans-freudenthal-building.
It is easy to get by bicycle to the summer school location. On the website of Utrecht Summer School or in the pre-departure information, which becomes available after you have paid the course fee, you can find information about renting bicycles and using the Utrecht public transport system.

The Mathematical Institute will provide lunches in the Mathematics Library, that is in room 707 on the 7th floor of the Hans Freudenthal building, on all ten days of the summer school, as well as dinners on Monday 10 August and Wednesday 19 August.

Saturday 8 August **Sunday 9 August**	**Key pick-up.** You will find the exact key pick-up location in the pre-departure information, which becomes available after you have paid the course fee.

Monday 10 August	**Álvaro del Pino Gómez: What is contact topology?** Consider the problem of modelling the motion of a coin rolling on a table. The coin can roll back and forth, and it can also twist the direction in which it points, but it cannot move sideways. This is a typical example of what we call “motion under constraints”, which is studied in Control Theory, an area of research between Mathematics and Engineering. In this mini-course we will learn about contact structures. These are the mathematical objects that allow us to discuss the example of the coin. Rather surprisingly, they pop up in many other settings: they are crucial in the mathematical formalization of optics and in Hamiltonian mechanics. Crucially for us, they also display a rich geometric/topological theory, making them an important object of study within Differential Topology. The goal of the mini-course is to develop some of this topological theory. We will make many pictures. **09:00** Registration **09:20** Welcome **09:30** Lectures and exercises **12:30** Lunch break **13:30-17:30** Lectures and exercises **17:30** We will walk to Theehuis Rhijnauwen, Rhijnauwenselaan 16 in Bunnik, for a pancake

Tuesday 11 August	Relinde Jurrius: Affine lines as division rings

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Relinde Jurrius: Affine lines as division rings

Forget everything you think you know about planar geometry. We will start again from scratch! An affine plane consists of a set, whose elements we call points, and a family of subsets of points, that we call lines, that satisfy three axioms:

1. For every pair of distinct points there is exactly one line that contains this pair of points.
2. For every line ℓ and for every point P not on ℓ, there is a unique line containing P that does not intersect ℓ.
3. Every line contains at least two points. There are at least two lines.

It is easy to see that the familiar Euclidean plane satisfies these properties. But there are many other affine planes. In particular, they can be finite. (Warm up exercise: what is the smallest affine plane?) We will see several examples of affine planes and prove some basic properties.

Our main goal is to prove the following theorem about affine lines.

> "Let ℓ be a line in a Desarguesian affine plane and let O and I be distinct elements of ℓ. Then ℓ is a division ring with O the identity element for addition and I the identity element for multiplication."

Again, with the Euclidean plane in mind, this theorem should not be a surprise: we can identify any line in the Euclidian plane with the number line. But it holds in a more abstract setting as well. Before proving this theorem, we need to introduce the context. In particular, what it means for an affine plane to be Desarguesian. Then we define, geometrically, addition and multiplication of points on an affine line. Finally, we show that these operations of addition and multiplication satisfies the properties needed to determine a division ring. Also, division rings will be defined and/or recapped.

If time permits, we will discuss how the main theorem can be used to coordinatize any Desarguesian plane by a division ring.

09:00 Lectures and exercises
12:30 Lunch break
13:30-17:00 Lectures and exercises

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<td>Wednesday 12 August</td>
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Lucas Dahinden: Catastrophic optics

A key phenomenon in optics is that a set of light rays is concentrated in some regions. For example, ideal lenses focus parallel light into a point. In most other situations, the focus happens in lines/surfaces, typically with singularities. A classic example is the reflection of light in a cup, forming a cardioid curve. In this mini-course, we learn a couple of analytic tools that can be used to explain and understand these more intricate foci. We start with the humble theory of envelopes that provide the first tool to transfer dynamics to geometry. We then refine the technique with generating functions. Finally, Arnold's catastrophe theory provides a classification of the singularities that can appear.

References.

- Catastrophy Optics: Morphologies of Caustics and their Diffraction Patterns. By M.V. Berry and C. Upstill, Progress in Optics XVIII, (1980), pp 257-346.
- Critical points of smooth functions and their normal forms. By V.I. Arnold. Russian Math. Surveys 30 (5) (1975), pp 1-75.
- Envelopes of lines, unfoldings and breaking symmetry. By P. Giblin and A. Wettig (2025) https://arxiv.org/abs/2506.16547.
- On cusps of caustics by reflection: billiard variations on the four vertex theorem and on Jacobi's last geometric statement. By G. Bor and S. Tabachnikov. Amer. Math. Monthly. 130 (2023), pp 454-467. Caustics Through the Looking Glass. By J. W. Bruce, P. J. Giblin, and C. G. Gibson. The Mathematical Intelligencer, 6 (1984), pp 47-58.
- Eliashberg's contributions towards the theory of generating functions. By Lisa Traynor, https://celebratio.org/Eliashberg_Y/article/1188.

09.00 Lectures and exercises
12.30 Lunch break
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Thursday 13 August	**Jan Hogendijk: Geometry and its applications before 1660** During the day there will be a few introductions by me but most of the time will be spent in interactive workshops. The morning session will mainly be devoted to the history of algebraic notation in geometry. It now seems self-evident to use such notations, but this was not always the case. After a brief introduction on sine computation and its uses, we begin with a workshop on the Dutch mathematician Ludolph van Ceulen, who in 1596 explained the computation of regular polygons and sines (without Taylor series!) by means of high-degree algebraic equations, in the so-called cossic notation, which he then solved by numerical algorithms. We will then work through part of the Geometry (1637) by Rene Descartes, who introduced the first system of algebraic notation in geometry close to the modern system. We will investigate the background of his algebraic system and argue that the effects of his mathematical work are more important than anything he had to say in philosophy. In the afternoon session we will turn to circle bundles, stereographic projection, and their manifestations and applications before 1660. First we will introduce the celestial sphere. Then participants will learn how to tell the time with a medieval astrolabe. If time and the enthusiasm of the participants allow, we will also learn how to read an Arabic astrolabe (knowledge of Arabic is not required). Finally, we will develop a proof in Greek style (and hence without algebraic notation at all) of the fact that stereographic projection maps circles on the (celestial) sphere to straight lines and circles in the plane. **09:00** Lectures and exercises **12:30** Lunch break **13:30-17:00** Lectures and exercises

Friday 14 August	**Soumya Sankar: Quaternion algebras and quadratic forms** Since their introduction in the 1800s, the Hamilton quaternions captured the imagination of pure and applied mathematicians alike, the latter owing primarily to their ability to describe rotations in space. Quaternion algebras are a generalization of the classical quaternions from the real numbers to arbitrary fields, and have found a variety of applications in algebra, number theory and both algebraic and hyperbolic geometry. In this course we will learn about quaternion algebras and explore their arithmetic and geometry. Underlying the algebraic aspects of quaternion algebras is the theory of quadratic forms, which we will also explore some higher dimensional aspects of. **09:00** Lectures and exercises **12:30** Lunch break **13:30-17:00** Lectures and exercises

Saturday 15 August **Sunday 16 August**	For the social programme organised by UU for all summer school students, see: https://utrechtsummerschool.nl/.

Monday 17 August	**Douwe Hoekstra and Sven Holtrop: Topological embeddings of compact n-dimensional metric spaces** In topology, many spaces we care about arise as subspaces of three-dimensional Euclidean space, such as the torus, sphere and Möbius strip. And in general, the n-torus and n-sphere live in $(n+1)$-dimensional Euclidean space. On the other hand, some spaces, such as the Klein bottle and real projective space, are not naturally realised as a subspace of Euclidean space. This raises the questions: what topological spaces can be embedded in Euclidean space, and what is the smallest dimension in which a space can be embedded? In this mini-course, we focus on compact n-dimensional metric spaces. We will start by introducing the somewhat unexpected definition of topological dimension. For example, it is remarkably hard to prove that the (vector space) dimension of Euclidean space and the topological dimension agree. With this notion, we can then show that the class of compact finite-dimensional metric spaces embed into Euclidean space. We will do this through exercises, guiding you through the proof, so that in the end, you can truly say you understand this theorem. **09:00** Lectures and exercises **12:30** Lunch break **13:30-17:00** Lectures and exercises

Tuesday 18 August	**Pieter Belmans: Quivers, representations, and classifications** Quivers are directed multigraphs, and they have a surprising application: they encode deep problems in linear algebra and generate interesting noncommutative algebras. We will build up the theory of quivers and their representations, and then consider various classification problems, of very different complexity. **09:00** Lectures and exercises **12:30** Lunch break **13:30-17:00** Lectures and exercises

Wednesday 19 August	**Frans Oort: The Last Entry by Gauss and elliptic curves** Carl Friedrich Gauss (1777-1855), already as young man, had many ideas. In his Tagebuch, which he started in 1796, he described these in 146 statements. In his Last Entry in July 1814 he made a conjecture about the number of solutions of a certain equation. We will analyze these 6 lines: what is the precise meaning, how did Gauss arrive at this amazing idea? We describe the geometry given by that equation and we will see that the theory of elliptic curves proves this conjecture. This visionary idea of Gauss is a prelude to later important developments: we show it is a special case of the Weil Conjectures, a central theme in 20-th century mathematics. Basic algebraic geometry and connections with number theory will be the core of this course. **09:00** Lectures and exercises **12:30** Lunch break **13:30-17:00** Lectures and exercises **18:00** Dinner at LE:EN, Heuveloord 140 in Utrecht

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<tr><td>Thursday 20 August</td><td>Bas Wensink: Complex analysis in several variables

Complex analysis in several variables is a major tool that has applications to both differential and algebraic geometry. In this course, we will see a brief introduction, highlighting some key results. We start with the basic definition of multivariable holomorphic functions, before moving on to some of the fundamental results: the holomorphic implicit function theorem, Hartogs's extension theorem and the Weierstrass preparation theorem. Afterwards, we will see some geometry appear by introducing affine analytic varieties and we will sketch a proof of Chow's theorem, a famous result that relates differential and algebraic geometry.

Prerequisites. Complex analysis in one variable (in particular, a good understanding of the Cauchy integral formula) and real analysis in several variables.

09:00 Lectures and exercises
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<tr><td>Friday 21 August</td><td>Viktor Blåsjö: Constructivism in the history of geometry

Concrete constructions are often a healthy supplement to abstract theory. Constructions can clarify meaning, produce empirically verifiable results, bring attention to hidden assumptions, and guard against paradoxes and inconsistencies. In this workshop, we reflect on the mathematical and philosophical significance of constructivism based on hands-on historical examples.

In the 17th century, instruments for drawing curves such as conic sections were an important topic, not least in the Netherlands among followers of Descartes. I have recreated a number of historical instruments using 3D-printing. We will try these out with our own hands and reflect on their purpose and role in the mathematics of their day.

Instruments and machines for drawing curves had been a prominent part of geometry already since antiquity. In particular, Euclid's ruler and compass signal his commitment to constructivism. To appreciate Euclid's vision of the foundations of mathematics, I provide participants with my "board game"-style edition of Book I of Euclid's Elements: a deck of cards with visualisations of Euclid's propositions and axioms, and a set of fill-in-the-blanks templates for an active and visual reading of Euclid's proofs, where the reader needs to find the right card to justify each logical step. I have developed this visual "board game" edition of The Elements to facilitate an active close reading of The Elements and a mathematically substantive and historically faithful engagement with this classic text.

09:30 Lectures and exercises
12:30 Lunch

N.B. You can store your luggage in room 611.</td></tr>
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