summerschool UTRECHT

Summer School on Geometry

11-22 August 2025

Last updated: May 7, 2025

Course organisers. Dr. Álvaro del Pino Gómez and Wilfred de Graaf MSc.
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For more information and to apply: www.utrechtsummerschool.nl/courses/science/geometry.

Location. Room 611 on the 6th floor of the Hans Freudenthal building, Budapestlaan 6, Utrecht: https://www.uu.nl/en/hans-freudenthal-building.
It is easy to get by bicycle to the summer school location. On the website of Utrecht Summer School or in the pre-departure information, which becomes available after you have paid the course fee, you can find information about renting bicycles and using the Utrecht public transport system.

The Mathematical Institute will provide lunches in the Mathematics Library, that is in room 707 on the 7th floor of the Hans Freudenthal building, on all ten days of the summer school, as well as a barbecue dinner and a pancake dinner.

Saturday 9 August **Sunday 10 August**	**Key pick-up.** You will find the exact key pick-up location in the pre-departure information, which becomes available after you have paid the course fee.

Monday 11 August	**Dr. Soumya Sankar and Hsin-Yi Yang MSc:** **Quaternion algebras and their applications** Since their introduction in the 1800s, the Hamilton quaternions captured the imagination of pure and applied mathematicians alike, the latter owing primarily to their ability to describe rotations in space. Quaternion algebras are a generalization of the classical quaternions from the real numbers to arbitrary fields, and have found a variety of applications in algebra, number theory and both algebraic and hyperbolic geometry. In this course, we will learn about quaternion algebras, see the arithmetic and geometry behind them, and explore some of their applications. **Prerequisites.** Groups, rings and fields. Knowing what a number field or an algebra is would be helpful, but not necessary. **09.00** Registration **09.20** Welcome **09.30** Lectures and exercises **12.30** Lunch break **13.30-17.30** Lectures and exercises

Tuesday 12 August	**Douwe Hoekstra MSc and Sven Holtrop MSc:** **Matrix Lie groups and Lie algebras** At the end of the 19th century, Sophus Lie and Friedrich Engel developed the theory of continuous transformation groups or Lie groups. These groups are locally described by a finite number of real parameters. Furthermore, these parameters carry some algebraic structure, which we call a Lie algebra nowadays. Most of the work of Lie focused on understanding the relation between Lie groups and algebras. One of the most celebrated results is Lie's 3rd theorem, which states that to any Lie algebra, we can associate a unique simply-connected Lie group. Throughout the past 130 years, Lie theory has become part of many branches of mathematics, including differential geometry, homotopy theory, algebraic geometry and even number theory. During this mini-course, we will take our first look at Lie theory through the so-called matrix Lie groups. During the morning, we will define matrix Lie groups and give examples. Then, we will continue defining Lie algebras and how to associate to any Lie group a Lie algebra. During the afternoon, we will focus on proving Lie's 3rd theorem in the context of matrix Lie algebras. **Prerequisites.** For this mini-course, we expect you to have a good grasp of topology and linear algebra. **Bibliography.** The main reference for this mini-course is chapters 1, 2, 3, and 5 of Lie Groups, Lie Algebras, and Representations by Brian C. Hall, which is available on the publisher's website. **09.00** Lectures and exercises **12.30** Lunch break **13.30-17.00** Lectures and exercises

Wednesday 13 August	**Prof. Dr. Frans Oort: Elliptic curves and congruent numbers** We discuss geometric methods in number theory. A definition and properties of elliptic curves will be given. We expect that this will be, for many students, their first introduction to algebraic geometry. An elliptic curve has the wonderful property that its points form a group. This combination of geometry, number theory, and algebra gives access to many applications. In particular, this will give methods to study the problem of congruent numbers that has been open for ten centuries. Many examples and exercises will be given. **09.00** Lectures and exercises **12.30** Lunch break **13.30-17.00** Lectures and exercises

Thursday 14 August	**Dr. Álvaro del Pino Gómez:** **In how many ways can we prove the Whitney-Graustein theorem?** A map of the circle into the plane is said to be immersed if its velocity is nowhere zero. This means that it traces a closed curve in the plane that has no creases, but may have self-intersections. In a 1937 article, Whitney attributes to Graustein the following result: two such immersions can be deformed to one another if and only if they "rotate the same". This is the first classification result for immersions of manifolds, a topic that eventually became one of the cornerstones of Differential Topology. The goal of this mini-course is to introduce you to a bunch of different techniques in the study of manifolds. Each technique will show up naturally as we pursue different strategies to prove the Whitney-Graustein theorem. **Prerequisites.** Analysis in multiple variables. Some background on topology (e.g. the fundamental group) is helpful but not strictly necessary. **09.00** Lectures and exercises **12.30** Lunch break **13.30-17.00** Lectures and exercises

Friday 15 August	**Christian Merten MSc and Raphael Douglas Giles MSc:** **A hands-on introduction to Formalizing Mathematics in Lean** An interactive theorem prover is a computer program to assist with the development of mathematical proofs. Formalizing mathematics is the process of typing mathematical proofs into the interface of an interactive theorem prover. In the past years, more and more mathematics, from undergraduate level to the frontiers of modern research have been formalized in a variety of proof assistants. In the recent AI for mathematics boom, proof assistants are an essential ingredient to prevent arbitrary hallucinations. In this course we will explore how formalizing mathematics works in practice in the proof assistant Lean using its mathematical library mathlib (https://github.com/leanprover-community/mathlib4). Based on examples and guided by the mathematical interests of the participants, we formalize proofs from the Bachelor curriculum. While in the morning, we go through the most important concepts of Lean, in the afternoon participants work on small formalization projects of their choice, assisted by experienced Lean users. **Prerequisites.** No familiarity with programming is required. We recommend to play a few worlds of the Natural Number Game (see https://adam.math.hhu.de/#/g/leanprover-community/nng4) in advance. **09.00** Lectures and exercises **12.30** Lunch break **13.30-17.00** Lectures and exercises

Saturday 16 August **Sunday 17 August**	For the social programme organised by UU for all summer school students, see: https://utrechtsummerschool.nl/.

Monday 18 August	Dr. Lucas Dahinden: Mathematical Billiards
	In Katok's words, Billiards is not a single mathematical theory, but rather a playground in which "various questions, conjectures, methods of solution are tested on billiard problems". In this course, we will spend time to marvel at some of the beautiful toys that this playground has to offer (Rainbows, bumper cars, ghosts, statistical dynamics etc.). On the theoretical side, we will focus on a variational principle: We will exploit geometric features to solve dynamical problems with the help of analytic tools. More specifically, we will prove the existence of periodic orbits in convex billiard tables (a Theorem of Birkhoff). **Prerequisites.** Mathematical analysis. **09.00** Lectures and exercises **12.30** Lunch break **13.30-17.00** Lectures and exercises

Tuesday 19 August	Dr. Gijs Heuts and Miguel Barata MSc: Differential topology from the point of view of homotopy theory
	09.00 Lectures and exercises **12.30** Lunch break **13.30-17.00** Lectures and exercises

Wednesday 20 August	Prof. Dr. Jan Hogendijk and Dr. Viktor Blåsjö: History of geometry
	The historical evolution of geometry provides a perspective on the subject that is not only mathematically but also philosophically, culturally, and pedagogically enriching. We begin by exploring Euclid's Elements through an interactive visual edition that we have developed. This classic text is intrinsically rewarding for its mathematical depth and beauty, and furthermore gives us occasion to reflect on timeless themes such as the role in mathematics of formalism, intuition, constructions, axiomatisation, and diagrammatic reasoning. We also briefly consider the wide-ranging cultural footprints of Euclid's text, which run across millennia of intellectual history. In the afternoon we will discuss the historical development of the objects that were studied in geometry. In Greek antiquity and the middle ages, geometry was limited to the study of straight lines, circles, conic sections, polyhedra, the sphere, cylinder, cone, and a few other curves and solids, which were all in some sense "constructible". In 1637 Descartes made a connection between geometry and algebra, and as a result algebraic curves were added to geometry, and their "construction" became less and less important over the years. Starting in the second half of the mid-nineteenth century, geometry was no longer limited to three dimensions and pre-existing objects such as the "plane" and "space". Mathematics gradually developed to the study of arbitrary structures invented by the human mind. **09.00** Lectures and exercises **12.30** Lunch break **13.30-17.00** Lectures and exercises

Thursday 21 August	**Dr. Paige North: An introduction to category theory** Category theory is a language that unites much of pure and applied mathematics. It shows up especially often in algebraic and geometric mathematics as well as logic and theoretical computer science. As a unifying language, it comes with tools that can be used and reused in a variety of different subjects. It also provides a setting in which to directly connect objects from different mathematical fields. This course will give an introduction to category theory, motivated by applications to algebra and topology. **Prerequisites.** None, but familiarity with abstract algebra and/or algebraic topology will help students appreciate the motivation. **09.00** Lectures and exercises **12.30** Lunch break **13.30-17.00** Lectures and exercises

Friday 22 August	**Bas Wensink MSc: Analysis in multiple complex variables** Complex analysis in one variable is in many ways different from real analysis in two variables. For instance, holomorphic functions are automatically analytic, bounded entire holomorphic functions are automatically constant, and contour integrals of holomorphic functions are invariant under homotopies. When considering several variables, there are a few more similarities with real analysis, but the theory is still vastly different. In this minicourse, we will introduce holomorphic functions of several variables and prove some basic results about them. The goal is to state and prove Hartogs' extension theorem, which highlights one of the key properties of zero sets of holomorphic functions. **Prerequisites.** Complex analysis in one variable (in particular, some familiarity with the Cauchy integral formula), real analysis in multiple variables. **09.30** Lectures and exercises **12.30** Lunch