Utrecht School for Mathematical Modeling *

Dates: 12-16 August 2024
Contact: Wilfred de Graaf w.f.degraaf@uu.nl
Organisers: Ivan Kryven, Palina Salanevich, and Wilfred de Graaf.

Apply at: utrechtsummerschool.nl

Location

The school will take place in Minnaert building, Room 2.08,
Adress: Leuvenlaan 4, 3584 CE Utrecht
Directions: 15 minutes from the city centre by public transport or bike. Information about renting bicycles can be found on the website of Utrecht Summer School.

*Updated: July 13

Meals and social program

The Mathematical Institute will provide lunches. The social program also includes:

- vegetarian barbecue on Monday evening;
- a dinner at Theehuis Rhijnauwen on Thursday;
- several optional activities. For example: a scavenger hunt (Tuesday), night kayaking or Pubquiz (Wednesday), visiting the botanical garden (Wednesday and Friday, before 16.30).

Saturday/Sunday, 10 and 11 August

Key pick-up. You will find the exact key pick-up location in the pre-departure information, which becomes available after you have paid the course fee.

Monday 12 August

- 09.00 - Welcome and registration
- 09.30 - Lectures and exercises: *See what's inside with X-rays - an introduction to the math behind computed tomography.*
- 12.30 - Lunch break
- 13.30 - Lectures and exercises: *Game theory*
- 17.30 - Barbecue dinner: we will walk along the Kromme Rijn river to *De Moestuin, Laan van Maarschalkerweerd 2*

See what's inside with X-rays - an introduction to the math behind computed tomography

Prof. dr. Tristan van Leeuwen

Soon after the discovery of X-rays by Wilhelm Röntgen in 1895, X-ray photographs were being used for medical diagnosis. In the 1970s the first CT-scanner was developed, employing X-rays to produce 3-dimensional images of human anatomy, facilitated by mathematical tools like the Fourier transform. Even today, this technique finds new applications, powered by novel measurement techniques and new developments in (computational) mathematics. In this mini-course I will give an overview of the mathematics behind CT, ranging from the Fourier and Radon transform, algebraic reconstruction techniques, compressed sensing, and the use of AI in imaging.

Keywords: Linear Algebra, Calculus, Inverse Problems in Imaging.

Playing with CATs: Using category theory to analyze games

Prof. dr. Willemien Kets

Game theory studies situations with strategic interdependence: There are multiple agents who each take decisions, and the outcome each agent obtains typically depends on the actions of others. Such analyses often lead to consideration of an infinite hierarchy of beliefs for each player. For example, Ann's choice of action will typically depend on what she believes Bob will do, but this in turn depends on what she believes that he believes that she will choose, and so on. Because such infinite hierarchies – sequences of probability measures on increasingly more complex spaces – are difficult to work with, mathematicians have developed a simple recursive method to model them. But are these recursive descriptions sufficiently expressive, i.e., can they model all possible belief hierarchies? We will address this question using the language of category theory, a beautiful theory of mathematical structures and their relations. The methods we develop can be used to study the expressiveness of recursive descriptions more broadly.

Tuesday 13 August

- 09.00 - 12:30 Lectures and exercises: *Introduction to Stochastic Modelling for Chemical and Biological Systems*
- 12.30 - Lunch break
- 13.30 - 17:00 Lectures and exercises: *Data dimension reduction with random matrices*

Data dimension reduction with random matrices

Dr. Sjoerd Dirksen

A common challenge in data analysis is dealing with high-dimensional data, such as pictures represented by many pixels or long gene sequences. The high dimensionality of this data makes it difficult to analyze or even store it. It is therefore of interest to convert a high-dimensional dataset into a low-dimensional one, while preserving some relevant properties of the original data. In this mini-course I will show that, surprisingly, random matrices can be an effective tool for dimension reduction – in fact, in certain settings they turn out to be optimal (in an appropriate sense) for this purpose, despite being random maps. I will present a detailed result in this direction and explain why it is useful for clustering data, i.e., the task of dividing data into subsets of similar objects.

Keywords: Probability, Data analysis, Dimension reduction.

Introduction to Stochastic Modelling for Chemical and Biological Systems

Dr. Chiheb Ben Hammouda

There are two fundamental approaches to the mathematical modeling of chemical and biological systems. Unlike deterministic models that rely on differential equations to predict future states, stochastic models accommodate randomness and are crucial for understanding systems where low molecular counts render deterministic approaches inaccurate or even inapplicable. These models are also relevant when observed phenomena depend on stochastic fluctuations, such as switching between two system states.

Assuming a basic familiarity with differential equations but no prior knowledge of advanced probability or stochastic analysis, this mini-course offers an accessible introduction to stochastic modeling and simulation algorithms for chemical and biological reactions, using illustrative examples. We also cover basic theoretical tools for analyzing these methods and discuss key distinctions and interconnections between stochastic and deterministic models. This highlights scenarios where deterministic models fall short, such as in systems that cannot be modeled by differential equations or in addressing questions beyond the reach of these models.

The focus of the mini-course is on the underlying mathematics and computational simulation; it does not require students to have taken any advanced courses in biology or chemistry. Moreover, the concepts learned can be extended to other applications, such as epidemiology and ecology.

Keywords: Probability, Differential Equations, Scientific Computing.

Wednesday 14 August

- 09.30 - Team contest: *μ-Games*
- 13.30 - Free time. *Tip: choose one of the activities from the social program, for example, night kayaking or pubquiz.*

μ-Games – a team activity involving algorithmic programming and clever problems

Organised by Niek Mooij

This will be a friendly programming contest focused on mathematical thinking. For all problems you must first think mathematically and then write a short program to calculate the answer. As soon as you complete your code it can be immediately cheeked by the server. You will be working on the problems in teams of 3 or 4 students. You can program in C#, Java, or Python. After the contest, we will conclude with a discussion where participants (and organisers) may share their solutions while having lunch.

Keywords: Recreational mathematics, Programming, Teamwork.

Thursday 15 August

- 09.00 - Lectures and exercises: *Graph-like behaviour in dynamical systems*
- 12.30 - Lunch break
- 13.30 - Exercises in virtual reality: *Graph-like behaviour in dynamical systems*
- 17.30 - We walk together to Theehuis Rhijnauwen (15min from the venue on foot) and have dinner there.

Graph-like behaviour in dynamical systems.

Dr. Ivan Kryven

After briefly revisiting the theory of linear stability for dynamical systems, we will look at several quadratic dynamical systems on a network. Specifically, we consider a set of ordinary differential equations (ODEs), some of which are coupled according to a given graph-like pattern. Although the dynamics of such systems are much more complex than those of linear ones, they can still be studied using basic tools in analysis. We will show that the asymptotic behaviour of these systems may reveal intriguing graph theoretical properties of the coupling network. For instance, we will show that finding the maximum clique in a graph — a well-known problem in discrete mathematics — can be reformulated as an asymptotic analysis problem for differential equations.

Keywords: Graph Theory, Differential Equations, Analysis

Friday 16 August

- 09.00 Lectures and exercises: *You are being framed! - An introduction to finite frame theory and its applications in data science*
- 12.15 Concluding words and summer school closing
- 12.30 Lunch break

You are being framed! - An introduction to finite frame theory and its applications in data science

Dr. Palina Salanevich

Basis is one of the most important concepts in mathematics. It allows to decompose an object into well-understood integral parts, just like a colour can be decomposed into its Red-Green-Blue components. Some of the properties of bases can be generalized to overcomplete systems of vectors called frames. Frames proved to be a powerful tool in many areas of applied mathematics, computer science, and engineering, as they provide a redundant, stable way of representing a signal.
The redundancy of a frame representation allows to recover a signal in the cases when part of the information is lost due to noise, quantization, erasures during transmission, or limitations of the measurement scheme. In this mini-course, we will learn the basics of the frame theory and see how the redundant frame representations can be used in different setups to solve various signal processing problems. During the practical part of the mini-course, you will work in groups on the problems related to the contemporary research in frame theory, including construction of frames with given properties, noise and erasure reduction, quantization, and others.

Keywords: Linear Algebra, Functional Analysis, Signal Processing.